Análise Não Linear de Pórticos Planos

CD - 1

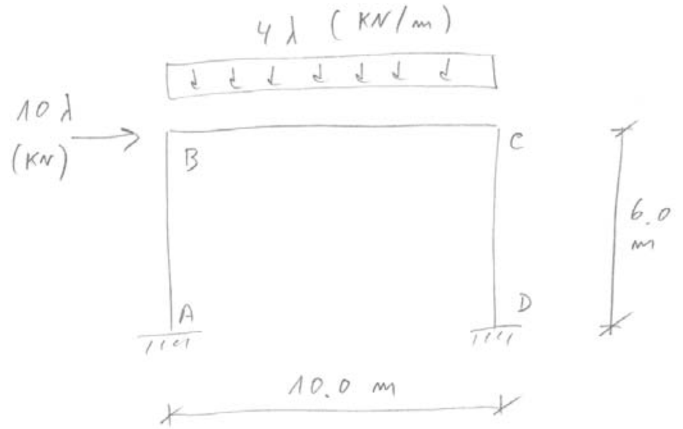
2005-12-08

Cargas distribuidas

Alvaro Azevedo

Exemplo

adaptado de (Ghali, Neville - 5ª Edição - pág. 591)



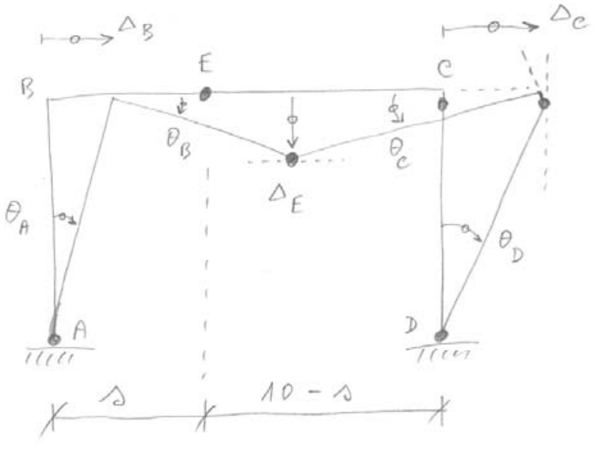
Pilares:

$M_p^P = 120$ KN m

Viga:

$M_p^V = 240$ KN m

Mecanismo combinado:



$$\Delta_C = \Delta_B = 6\,\theta_A$$

$$\theta_B = \theta_A$$

$$\theta_D = \theta_A$$

$$\Delta_E = s\,\theta_B = s\,\theta_A$$

$$\theta_C = \frac{\Delta_E}{10-s} = \frac{s\,\theta_A}{10-s}$$

CD - 2

Cálculo de $\lambda(s)$:

Trabalho externo = Trabalho interno

$$10\lambda\,\Delta_B + 4\lambda s\,\frac{\Delta_E}{2} + 4\lambda(10-s)\,\frac{\Delta_E}{2} =$$

$$= M_p^P\,\theta_A + M_p^V\,\theta_B + M_p^V\,\theta_C + M_p^P\,\theta_C + M_p^P\,\theta_D + M_p^P\,\theta_D$$

$$10\lambda\,6\theta_A + 4\lambda s\,\frac{s\theta_A}{2} + 4\lambda(10-s)\,\frac{s\theta_A}{2} =$$

$$= 120\,\theta_A + 240\,\theta_A + 240\,\frac{s\,\theta_A}{10-s} + 120\,\frac{s\theta_A}{10-s} + 120\,\theta_A + 120\,\theta_A$$

$$60\lambda + \cancel{2\lambda s^2} + 20\lambda s - \cancel{2\lambda s^2} = 600 + \frac{360\,s}{10-s}$$

$$(60 + 20s)\lambda = \frac{6000 - 240s}{10-s}$$

$$\lambda = \frac{6000 - 240s}{600 - 60s + 200s - 20s^2} = \frac{6000 - 240s}{600 + 140s - 20s^2}$$

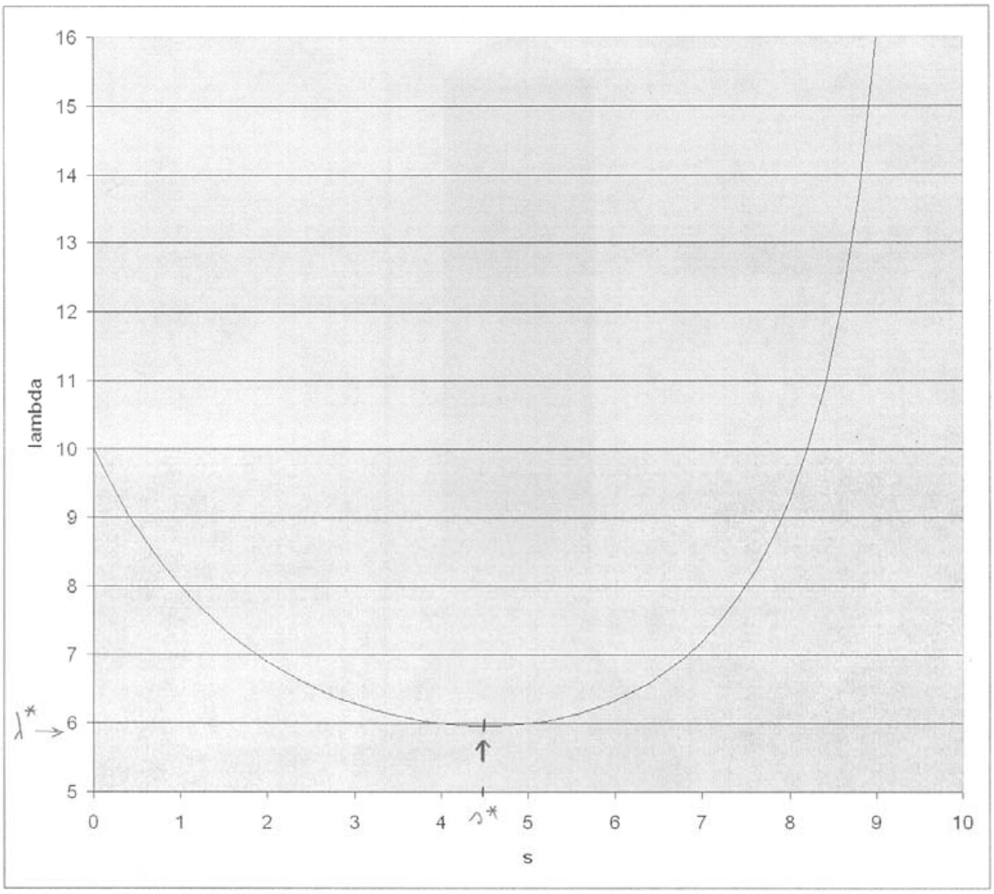
$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$\frac{d\lambda}{ds} = \frac{(-240)(600+140s-20s^2) - (6000-240s)(140-40s)}{(600+140s-20s^2)^2} = 0$$

$s^* = 4.506\ 098$ m

$\lambda^* = 5.963\ 631 \rightarrow$ valor mínimo de $\lambda(s)$

CD-3

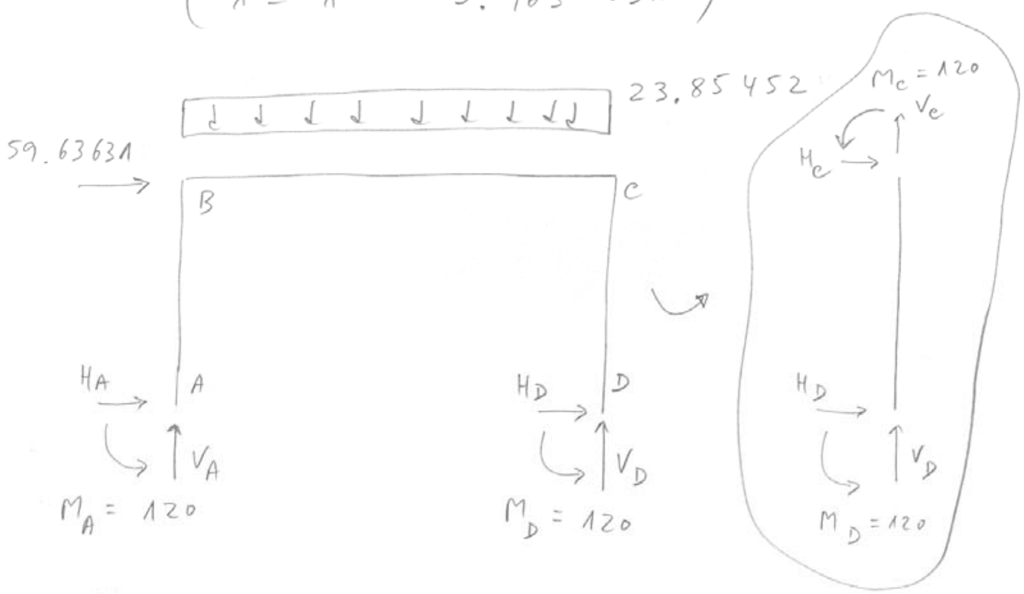


grafico_s_lambda.xls

CD - 4

Diagrama de momentos final

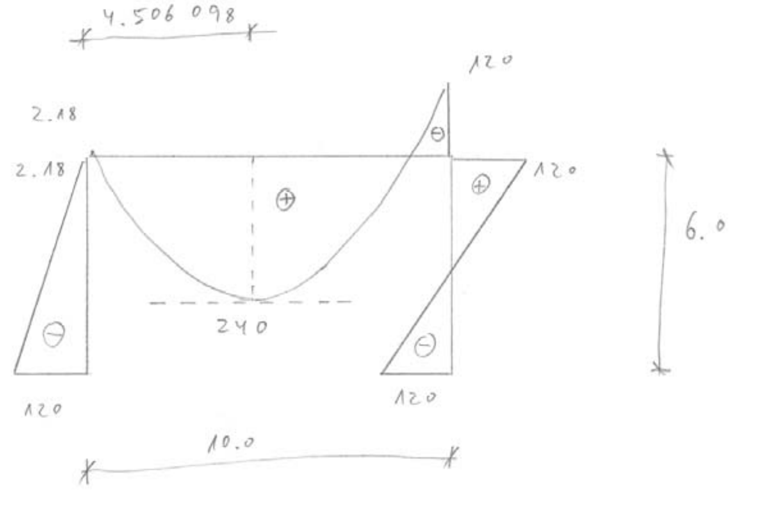
$$(\lambda = \lambda^{*} = 5.963\ 631)$$



$$\sum M_C = 0 \Rightarrow 2 \times 120 + 6\,H_D = 0 \Rightarrow H_D = -40 \text{ KN}$$
(Barra CD)

$$\sum F_H = 0 \Rightarrow H_A + H_D + 59.63631 = 0 \Rightarrow H_A = -19.63631 \text{ KN}$$
(total)

$$M_B = -120 - 6H_A = -2.18214 \text{ KNm}$$



EEA - 1

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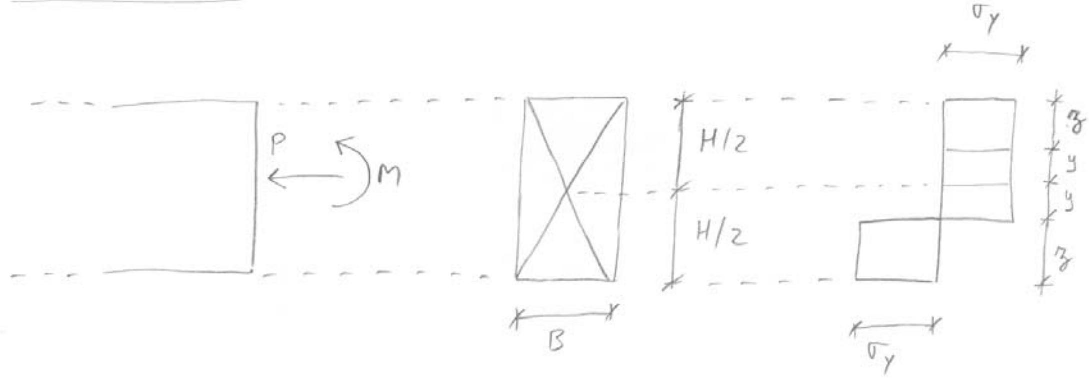
Alvaro Azevedo

Efeito do esforço axial



$$z = \frac{H}{2} - y$$

$$P = 2\,y\,B\,\sigma_y \Rightarrow \boxed{P = 2\,\sigma_y\,B\,y}$$

$$M = 2\left[z\,B\,\sigma_y\left(y + \frac{z}{2}\right)\right]$$

$$M = 2\left[\left(\frac{H}{2} - y\right) B\,\sigma_y \left(y + \frac{H}{4} - \frac{y}{2}\right)\right]$$

$$M = 2\,B\,\sigma_y\left(\frac{Hy}{2} + \frac{H^2}{8} - \frac{Hy}{4} - y^2 - \frac{Hy}{4} + \frac{y^2}{2}\right)$$

$$M = 2\,B\,\sigma_y\left(\frac{H^2}{8} - \frac{y^2}{2}\right)$$

$$\boxed{M = \sigma_y\,B\left(\frac{H^2}{4} - y^2\right)}$$

EEA-2

$$\begin{cases} P = 2\,\sigma_y\, B\, y \\ M = \sigma_y\, B \left(\frac{H^2}{4} - y^2 \right) \end{cases}$$

$$y = 0 \Rightarrow P = 0 \quad ; \quad M = \sigma_y \frac{BH^2}{4}$$

$$y = \frac{H}{2} \Rightarrow P = \sigma_y B H \quad ; \quad M = 0$$

$$\bar{M} = \sigma_y \frac{BH^2}{4} \quad ; \quad \bar{P} = \sigma_y B H$$

$$M = \sigma_y B \left(\frac{H^2}{4} - y^2 \right) \quad ; \quad P = 2\,\sigma_y B y$$

$$\frac{M}{\bar{M}} = \frac{\frac{H^2}{4} - y^2}{\frac{H^2}{4}} = 1 - \frac{4y^2}{H^2} = 1 - \left(\frac{2y}{H} \right)^2$$

$$\frac{P}{\bar{P}} = \frac{2y}{H}$$

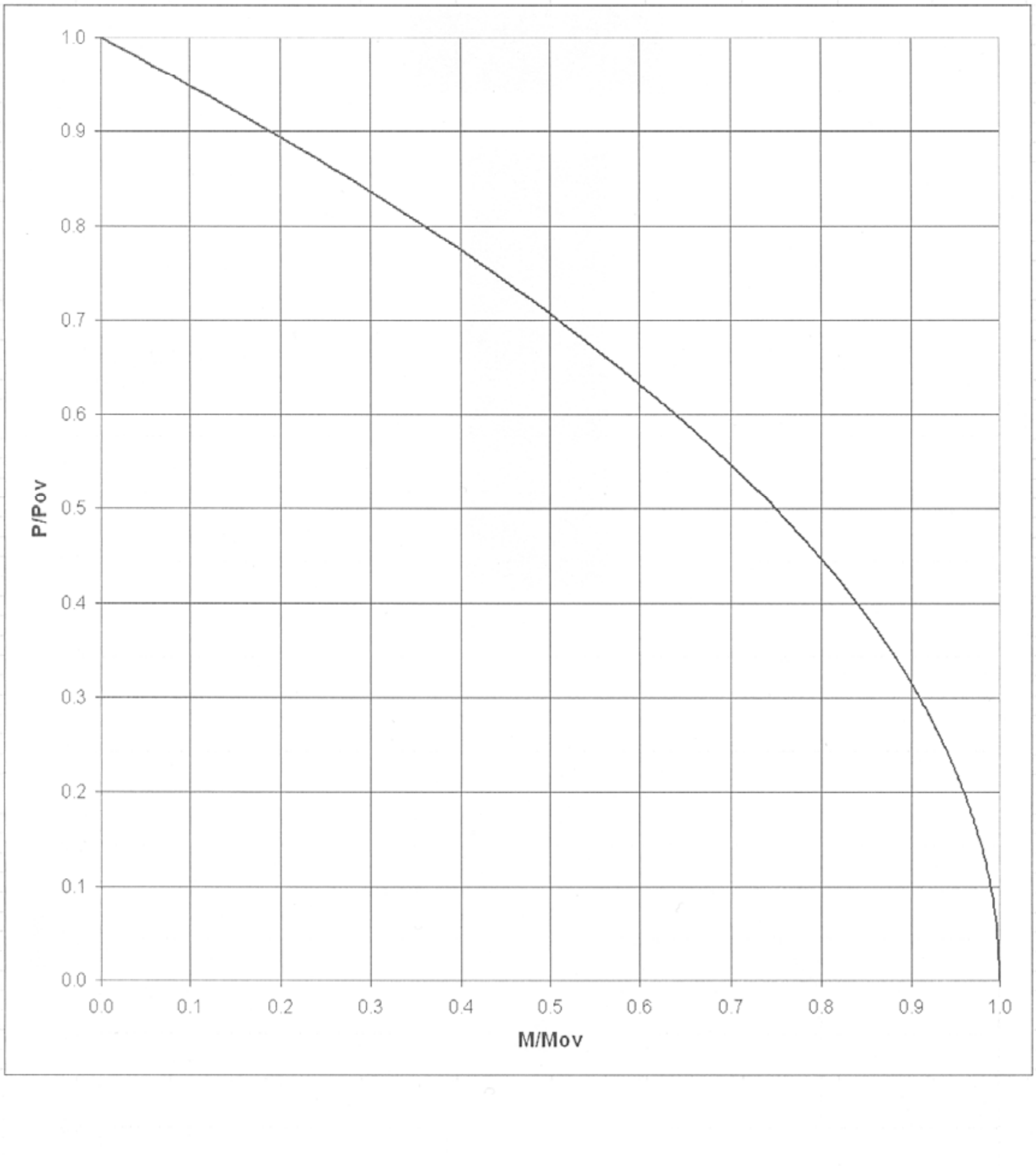
$$\frac{M}{\bar{M}} = 1 - \left(\frac{P}{\bar{P}} \right)^2$$

$$\frac{P}{\bar{P}} = \sqrt{1 - \frac{M}{\bar{M}}}$$

EEA-3

File: grafico_M_P.xls

Date: 2005-12-12



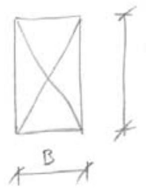
Optimização de Pórticos Planos em Regime Não Linear

OPP-1

2005-12-08

Álvaro Azevedo

B. G. Neal
The Plastic Methods of Structural Analysis
3rd Edition, Capítulo 7
Chapman and Hall, 1977



$B = cH$ (sendo c fixo)

$A = BH = cH^2$

$$M_p = \sigma_y \frac{BH^2}{4} = \sigma_y \frac{cH^3}{4} = \frac{\sigma_y c}{4} H^3$$

$$H^3 = \frac{4M_p}{\sigma_y c} \Rightarrow H^2 = \left(\frac{4}{\sigma_y c}\right)^{2/3} M_p^{2/3}$$

$$A = c\left(\frac{4}{\sigma_y c}\right)^{2/3} M_p^{2/3} = \left(\frac{4}{\sigma_y}\right)^{2/3} \frac{c}{c^{2/3}} M_p^{2/3} = \left(\frac{4}{\sigma_y}\right)^{2/3} c^{1/3} M_p^{2/3}$$

$p \rightarrow$ peso da barra por unidade de comprimento

$$p = \gamma A$$

$$p = \gamma \left(\frac{4}{\sigma_y}\right)^{2/3} c^{1/3} M_p^{2/3}$$

$P \rightarrow$ peso da estrutura (barras de secção constante)

$$P = \sum_{i=1}^{NB} p_i L_i = \sum_{i=1}^{NB} \left[\gamma \left(\frac{4}{\sigma_y}\right)^{2/3} c^{1/3} M_{pi}^{2/3} L_i\right]$$

$$P = \gamma \left(\frac{4}{\sigma_y}\right)^{2/3} e^{1/3} \sum_{i=1}^{NB} \left[M_{p_i}^{2/3} \; L_i \right]$$

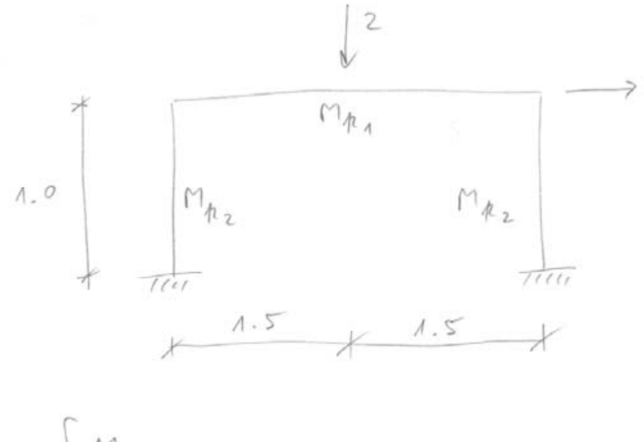
Minimizar P é o mesmo que minimizar

$$P' = \sum_{i=1}^{NB} \left[L_i \; M_{p_i}^{2/3} \right]$$

De um modo simplificado pode-se considerar

$$P' \simeq \sum_{i=1}^{NB} \left[L_i \; M_{p_i} \right]$$

Exemplo :



Calcular a solução (M_{p1}^{*}, M_{p2}^{*}) à qual corresponde um peso total mínimo

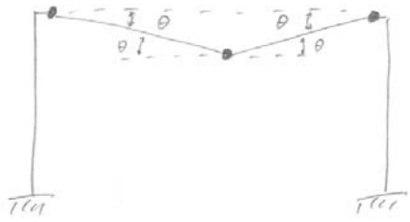
$$\begin{cases} M_{p1} \rightarrow x_1 \\ M_{p2} \rightarrow x_2 \end{cases}$$

$\boxed{M_{p1} \le M_{p2}}$ $(\nu_1 \le \nu_2)$

$\boxed{M_{p1} \ge M_{p2}}$ $(\nu_1 \ge \nu_2)$

OPP-3

(A)

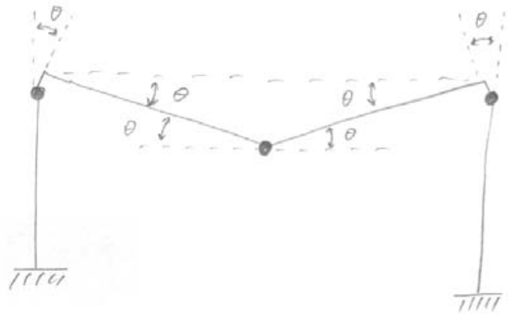


$$2 \times 1.5\,\theta = 4 M_{p1}\,\theta$$

$$4\nu_1 - 3 = 0$$

$$\nu_1 \ge 3/4$$

(B)



$$2 \times 1.5\,\theta = 2 M_{p1}\,\theta + 2 M_{p2}\,\theta$$

$$2\nu_1 + 2\nu_2 - 3 = 0$$

$$\nu_2 \ge -\nu_1 + 3/2$$

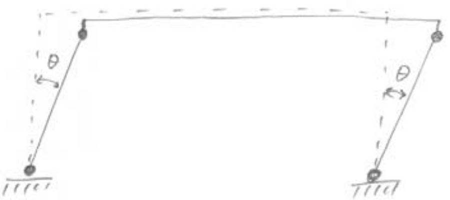
(C)

$$1 \times 1 \times \theta = 2 M_{p1}\,\theta + 2 M_{p2}\,\theta$$

$$2\nu_1 + 2\nu_2 - 1 = 0$$

$$\nu_2 \ge -\nu_1 + 1/2$$

(D)

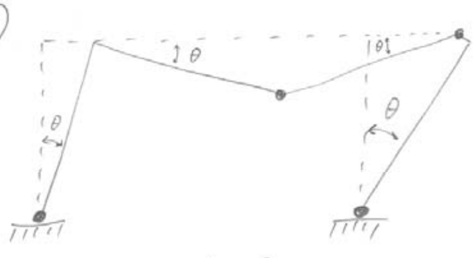


$$1 \times 1 \times \theta = 4 M_{p2}\,\theta$$

$$4\nu_2 - 1 = 0$$

$$\nu_2 \ge 1/4$$

(E)



$$2 \times 1.5\,\theta + 1 \times 1 \times \theta =$$

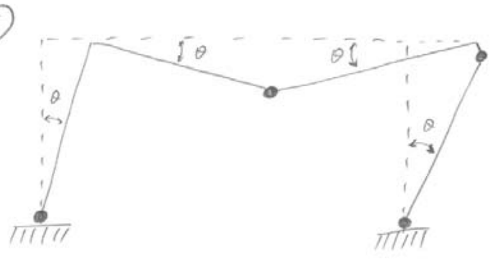
$$= M_{p2}\,\theta + 2 M_{p1}\,\theta + 2 M_{p1}\,\theta + M_{p2}\,\theta$$

$$4 = 4\nu_1 + 2\nu_2$$

$$2\nu_1 + \nu_2 - 2 = 0$$

$$\nu_2 \ge -2\nu_1 + 2$$

(F)



$$2 \times 1.5\,\theta + 1 \times 1 \times \theta =$$

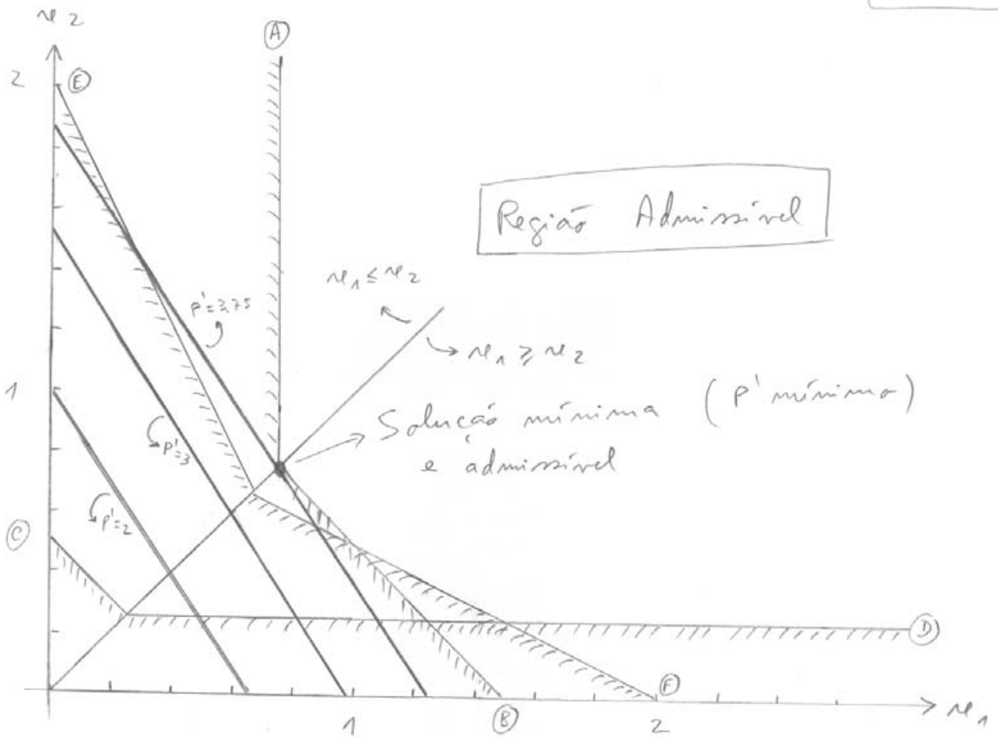
$$= M_{p2}\,\theta + 2 M_{p1}\,\theta + 2 M_{p2}\,\theta + M_{p2}\,\theta$$

$$4 = 2\nu_1 + 4\nu_2$$

$$\nu_1 + 2\nu_2 - 2 = 0$$

$$\nu_2 \ge -\frac{1}{2}\nu_1 + 1$$

OPP - 4



Minimizar:

$$P' = 3x_1 + 2x_2 \quad \begin{cases} 3x_1 + 2x_2 = 2 \Rightarrow x_2 = -\frac{3}{2}x_1 + 1 \\ 3x_1 + 2x_2 = 3 \Rightarrow x_2 = -\frac{3}{2}x_1 + \frac{3}{2} \\ 3x_1 + 2x_2 = 3.75 \Rightarrow x_2 = -\frac{3}{2}x_1 + 1.875 \end{cases}$$

$$\text{Solução} = (x_1^*, x_2^*) = (0.75, 0.75)$$

Intersecção das retas A e B

$$\begin{cases} M_{p_1}^* = 0.75 \\ M_{p_2}^* = 0.75 \end{cases}$$