Truss Sizing and Shape Optimization: A Second-Order Approach

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Abstract

In most structural optimization applications only the first derivatives of the objective function and constraints are used by the numerical algorithm. Some authors advocate the use of derivative free genetic algorithms due to the simplicity of their implementation and their ability to solve problems with multiple local minima and discontinuities. When first and second derivatives can be easily obtained their use often leads to an increased accuracy, convergence rate and reliability of the optimization process. This last approach is presented in this work, with applications to the sizing of truss members and to the shape optimization of a steel bridge.

The optimization of a structure can be formulated as a mathematical program whose objective function is the cost and whose constraints define the structural behavior and bound the stresses and the displacements at some selected locations. When the displacement method is used, the functions that define the mathematical program are explicit functions of the design and behavior variables. First and second derivatives of those functions can be symbolically calculated by the computer algorithm in an efficient and accurate way. In the examples presented here all the expressions are polynomials with positive or negative coefficients and exponents. The derivation of those functions is straightforward and can be easily extended to the determination of higher order derivatives. Some work under progress will extend these capabilities to other types of functions.

These ideas were implemented in a computer program named NEWTOP. The mathematical program is defined by an objective function and by inequality and equality constraints. These functions are supplied in a data file that is parsed by the computer program. Each inequality constraint is converted into an equality constraint by the addition of a squared slack variable. Since now there are only equality constraints, the Lagrangian function may be defined and a set of Lagrange multipliers is introduced. With the exception of some rare situations, the solution of the mathematical program is a stationary point of the Lagrangian. This necessary condition leads to a system of nonlinear equations whose solution is

calculated by the Newton method. In each Newton iteration a system of linear equations has to be solved. Its coefficient matrix is the Hessian of the Lagrangian and the right-hand side vector is the gradient of the Lagrangian. This system of linear equations is solved by a direct Gaussian elimination method or by an iterative conjugate gradient method. In both cases the sparsity pattern of the Hessian is taken into account leading to large economies in terms of storage requirements and computation time. All the derivatives that are required to build the Hessian matrix and the gradient of the Lagrangian are symbolically determined as described above. These derivatives are efficiently calculated and their accuracy is much higher than in the case of numerical differentiation. In order to improve the convergence characteristics of the Newton method scaling techniques are used. The variables are scaled by a factor that depends on the initial solution and the constraints are normalized, i.e., they are multiplied by a factor that causes the gradient of each constraint to be equal to the gradient of the objective function. The normalizing coefficients are also based on the initial solution. For these reasons the initial solution must be carefully chosen. It is also well known that the success of the Newton method is highly dependent on the initial solution. In structural optimization problems it is usually easy to obtain by traditional methods a solution that is both feasible and sufficiently close to the optimal solution. From such a starting point the Newton method is expected to exhibit quadratic convergence. The selection of the value of the line search parameter is also critical and strongly influences the convergence path. In some problems the value that minimizes the error in the Newton direction is the best choice while in others a trust region like approach leads to a more stable convergence that in some situations may be decisive. During the iteration process it is possible to visualize the evolution of some parameters and to draw a histogram with the variation of the scaled variables. This information may suggest that some parameters or the convergence strategy have to be changed. This user intervention may be performed with no need to restart the iteration process.

This algorithm has been used to solve some optimization problems, e.g., truss sizing, shape optimization of trusses, sizing of frames with nonlinear behavior, etc. In truss sizing problems the variables are the cross-sectional areas of the members, equality constraints are the equilibrium equations in each degree of freedom and the inequality constraints are bounds in stresses, displacements and cross-sectional areas. Local buckling constraints may also be considered. Problems with more than 4 000 independent design variables and 20 000 constraints have been successfully solved. In shape optimization problems the coordinates of some nodes may also change. This approach has been used to optimize the shape of a steel bridge.