

Implementation of a Three-Dimensional Time Domain Formulation for the Prediction of Vehicle Induced Vibrations

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Abstract

The implementation and validation of a numerical analysis methodology which can be used in the prediction of vehicle induced vibrations is described. This methodology is formulated in the time domain for the three-dimensional case. Although presented in the context of vehicle induced vibrations, it can be adopted in the study of general problems of dynamic soil-structure interaction in which the structure lies on the surface of the soil. The method is based on the coupling of the fundamental Green functions to a finite element code (FEMIX). These functions describe the behaviour of the unbounded soil, while the use of finite elements enables an adequate simulation of the part of the domain where inhomogeneities or nonlinearities occur. Some details about the implementation are discussed and two numerical examples are presented: the analysis of the dynamic response of a rigid square surface foundation and the study of the dynamic response of a slab resting on a homogeneous halfspace and subjected to the passage of a moving load.

Keywords: vehicle induced vibrations, dynamic soil-structure interaction, three-dimensional numerical analysis, time domain, finite element method, Green functions.

1 Introduction

In recent years a reasonable effort has been undertaken to develop numerical methods for the study of problems involving a dynamic soil-structure interaction. The simulation of the phenomenon of vehicle induced vibrations clearly requires an adequate consideration of the dynamic track-soil or road-soil interaction. The expansion of the high-speed railway lines contributed to the relevance of this kind of studies.

The Finite Element Method (FEM) is a very popular and attractive tool due to its versatility. However, due to the semi-infinite nature of the soil, this method presents some limitations when used in the simulation of this type of problems. Since it is not possible to fully discretize the soil, some type of domain truncation is required. These artificial boundaries lead to spurious reflections, which affect the behaviour of the modelled part of the system. When the discretized domain is very large (in order to move the artificial boundaries away from the part of the model with interest for the study), the problem becomes computationally intractable. The problems associated with these spurious reflections may be minimized (but not fully eliminated) using absorbing boundaries (application of viscous dampers connected to the boundaries of the finite element model) [1], Perfectly Matched Layers (PML) [2] or infinite elements [3].

As an alternative, some authors applied hybrid methods combining the Finite Element Method and the Boundary Element Method (FEM/BEM) in order to take advantage of the versatility of FEM and the capability of the BEM to simulate infinite media [4].

Other authors developed numerical models based on a 2.5D dimension concept, e.g., 2.5D finite/infinite elements (FEM/IEM) [5] and 2.5D FEM/BEM [6]. These procedures are very efficient from the computational point of view. The main drawback of these models is associated with the assumption of invariability of the geometry and properties of the structure along one direction. Usually they are also limited to linear or equivalent linear analyses.

The present work refers the implementation and verification of a 3D numerical methodology formulated in the time domain and based on the coupling of the fundamental Green functions to a FE code [7, 8]. These functions reproduce the behaviour of the soil while the FE model can simulate a complex structure. The advantages of this procedure are related to the possibility of modelling structures with complex geometries which are not invariant along any direction, as well as the capability of performing nonlinear analyses. The main drawback of these models is their high cost in terms of time and computational resources.

2 Methodology

In this work the implementation of a numerical analysis methodology which can be used in the prediction of train induced vibrations is described [7]. Although it is presented in the context of the soil-track interaction, it can be adopted in the study of general three-dimensional problems of dynamic soil-structure interaction (SSI), when the structure lies on the surface of the soil. This procedure is formulated in the time domain and enables the consideration of complex geometries and nonlinearities in the simulation of the behaviour of the structure.

According to the presented methodology, the system is divided into two independent substructures: a) the superstructure; b) its supporting soil. The superstructure is studied with the Finite Element Method (FEM) while the soil behaviour is modelled by means of the fundamental Green Functions of a halfspace or a layered halfspace with linear elastic behaviour. The soil surface is assumed to be plane and horizontal. The coupling between the structure and the supporting soil is established at the interaction surfaces by means of equilibrium and compatibility equations.

A brief description of the implemented procedure is presented below. A more detailed information is available in [8].

2.1 Description of the superstructure (Finite Element Method)

The superstructure includes all parts of the system that are located above the soil, being its behaviour simulated by means of the FEM. Thus it is possible to consider complex geometries, heterogeneities and nonlinearities. The dynamic equation of the structure can be written as

$$\mathbf{M} \ddot{\mathbf{u}} + \mathbf{C} \dot{\mathbf{u}} + \mathbf{K} \mathbf{u} = \mathbf{P} \quad (1)$$

where \mathbf{M} , \mathbf{C} and \mathbf{K} are the mass matrix, damping matrix and stiffness matrix of the superstructure, \mathbf{u} is the displacement vector and \mathbf{P} is the vector of external forces. A dot over a variable denotes differentiation with respect to time.

2.2 Description of the soil (Green Functions)

As referred above, the soil behaviour is described by means of the fundamental Green Functions, which define the dynamic response of the soil due to an unit load applied at any point. The presented tool contemplates the possibility of considering the soil as a homogeneous halfspace or as a layered halfspace with linear elastic behaviour.

The presented procedure is based on the following assumptions:

- the continuous time histories of the soil pressures are approximated by a sequence of rectangular pulses whose value corresponds to the average of the initial and final values of the respective time step;
- the soil-structure interaction surface is discretized with interaction elements and a uniform pressure is assumed within each element. This division is defined in accordance with the finite element mesh in such a way that each interaction element corresponds to a face of a finite element contacting with soil.

As a consequence of these assumptions the dynamic response of the soil due to pressure loads (applied in the surface associated with the interaction elements) with

rectangular time-dependence must be calculated. This response may be obtained by subtracting the responses due to a pair of loads with Heaviside time-dependence applied at consecutive time steps.

The displacements at the surface of the soil due to a point load with Heaviside time-dependence applied at the surface of the soil can be calculated analytically in time-space domain for the case of a homogeneous halfspace [9]. However, these analytical solutions neglect the effect of damping and are limited to some values of the Poisson's ratio.

When dealing with a layered halfspace, the response of the soil is computed numerically using the Direct Stiffness Method available in the Elastodynamics Toolbox for MATLAB - EDT [10, 11]. In this case the Green functions are calculated in the frequency-wavenumber domain and are then transformed to time-space domain.

As described above, this methodology requires the calculation of the soil response due to a pressure load with Heaviside time-dependence. In the cases where the soil response is calculated analytically, the solution due to a pressure load is obtained by spatially integrating the solution due to a point load. When the Green functions are calculated in the frequency-wavenumber using EDT, the desired response can be obtained using a more efficient procedure that consists on the multiplication of the solution by the wavenumber content of the loaded area and by the frequency content of a Heaviside load before the transformation to the time-space domain. According to the considerations described above and to

$$\mathbf{w}^j = \mathbf{H}^j \mathbf{q} \quad (2)$$

the matrix \mathbf{H}^j ($\mathbf{H}^{j<1} = \mathbf{0}$) is used to calculate the soil displacement vector \mathbf{w}^j observed at time step $t^j = j \Delta t$ due to an interface pressure \mathbf{q} applied at the time $t = 0$ and kept constant (Heaviside time dependence).

Considering the matrix \mathbf{F}^k defined as

$$\mathbf{F}^k = \frac{(\mathbf{H}^{k+1} - \mathbf{H}^{k-1})}{2} \quad (3)$$

the Equation (2), which relates the soil displacements at a specific time step with the time evolution of the soil pressures, can be written as

$$\mathbf{w}^i = \sum_{j=2}^i (\mathbf{F}^{j-1} \mathbf{q}^{i-j+1}) + \mathbf{F}^0 \mathbf{q}^i \quad (4)$$

2.3 Soil-structure coupling

As referred above, the coupling is established at the interaction surfaces by enforcing the equilibrium of forces and making the displacements compatible.

In this work, it is imposed that the displacements at each interaction point (soil) are compatible with the displacements at the corresponding point of the finite element model. The relation between the nodal displacements of the finite elements, \mathbf{u} , and the displacements of the interaction points (superstructure), \mathbf{v} , can be written as

$$\mathbf{v} = \mathbf{T}_u \mathbf{u} \quad (5)$$

where \mathbf{T}_u represents a transformation matrix whose terms result from the finite element shape functions evaluated at the interaction points.

Thus, the displacements are made compatible by imposing

$$\mathbf{w} = \mathbf{v} \quad (6)$$

On the other side, the equilibrium of forces at the interaction surface is guaranteed by imposing that the action of the soil on the structure has the same magnitude as the action of the structure on the soil and opposite signal. Thus, it is necessary to calculate the nodal interaction forces, \mathbf{Q} , corresponding to the soil pressures, \mathbf{q} :

$$\mathbf{Q} = -\mathbf{T}_q \mathbf{q} \quad (7)$$

where \mathbf{T}_q represents a transformation matrix which results from the integration of the finite element shape functions over the area of the interaction elements.

Adding the interaction forces, \mathbf{Q} , to Equation (1) one has

$$\mathbf{M} \ddot{\mathbf{u}}^i + \mathbf{C} \dot{\mathbf{u}}^i + \mathbf{K} \mathbf{u}^i = \mathbf{P}^i - \mathbf{T}_q \mathbf{q}^i \quad (8)$$

Taking into account (4) and performing some mathematical work, Equation (8) can be written as:

$$\mathbf{M} \ddot{\mathbf{u}}^i + \mathbf{C} \dot{\mathbf{u}}^i + (\mathbf{K} + \mathbf{K}^{act}) \mathbf{u}^i = \mathbf{P}^i + \mathbf{Q}^{hist} \quad (9)$$

where

$$\mathbf{K}^{act} = \mathbf{T}_q (\mathbf{F}^0)^{-1} \mathbf{T}_u \quad (10)$$

and

$$\mathbf{Q}^{hist} = \mathbf{T}_q (\mathbf{F}^0)^{-1} \sum_{j=2}^i (\mathbf{F}^{j-1} \mathbf{q}^{i-j+1}) \quad (11)$$

The influence of the supporting soil in the behaviour of the system is represented by means of the dynamic stiffness matrix of the soil \mathbf{K}^{act} and the vector of the historical interaction forces \mathbf{Q}^{hist} . Since full contact is assumed (see Equation (6)), the stiffness matrix \mathbf{K}^{act} is calculated and added to the stiffness matrix of the structure only once before the first time step. On the other hand, as the vector \mathbf{Q}^{hist} changes with time, it has to be calculated and added to the system of equations before processing each time step. The dynamic equation of the system, defined by (9), can be solved by a classical time step integration procedure as the Newmark Method.

3 Some comments about the implemented methodology

3.1 Symmetry of the stiffness matrix of the soil

According to [7], when all the interaction elements are identical and the interaction stresses are assumed to be uniform within each element, the matrix \mathbf{F}^0 is symmetric. However, these conditions do not guarantee the symmetry of the stiffness matrix of the soil \mathbf{K}^{act} . Thus, it is possible to conclude that \mathbf{K}^{act} is symmetric only when \mathbf{F}^0 is symmetric and simultaneously the following condition is ensured

$$\mathbf{T}_q^T = c \mathbf{T}_u \quad (12)$$

In this work, Equation (12) is only valid when the elements involved in the interaction are 8-node solid elements.

3.2 Stability of the process

In the developed tool, the dynamic equation of the system is solved by the Newmark Method. In fact, when adequate values of its parameters are used the method is stable, i.e., the stability of the procedure does not depend on the discretization adopted for time and space. However, the coupling of the structure and the supporting soil using interaction elements changes this scenario. In order to avoid instabilities associated with the interaction elements, the time step must be chosen in accordance with the spatial discretization at the interaction surfaces. Several authors identified this phenomena [7, 12] and some of them suggest reference time steps, which do not guarantee the stability of the process. The use of smaller time steps may lead to instabilities. In the analyses whose results are presented in section 4 the adopted time step, Δt , takes into account the following criterion [7]:

$$\Delta t = \frac{d}{2 C_R} \quad (13)$$

where d is the diagonal of the interaction elements and C_R is the velocity of the surface waves of the soil.

3.3 Response of points on the soil surface

It is important to note that, knowing the time history of the interaction stresses, the calculation of the response on the surface soil is very simple. In fact, the displacement of a point ξ^r on the surface of the soil at the instant $t^j = j \Delta t$ is given by

$$w_{\xi^r}(j \Delta t) = \sum_{i=1}^j F_{\xi^r}^{j-i} q^i \quad (14)$$

where $F_{\xi^r}^k$ is obtained as described in section 2.2.

4 Numerical examples

In this section two numerical examples are presented, corresponding the former to the analysis of the dynamic response of a rigid square surface foundation and the latter to the study of the dynamic response of a slab resting on a homogeneous halfspace due to a passage of a moving load. The developed tool can be used in the prediction of vibrations induced by trains (in the track as well as in the surface of the adjacent soil), corresponding the presented tests to a part of the validation procedure based on several steps of increasing complexity.

4.1 Dynamic response of a rigid square foundation

As a first example, the study of the dynamic response of a rigid square surface foundation resting on a homogeneous halfspace is considered. This example has been chosen due to the simplicity of the studied structure as well as to the possibility of comparing the obtained results with those presented in the literature [7]. In order to facilitate this comparison, the geometry and material properties presented in Figure 1 are used for the foundation. The rigidity of the foundation is simulated by adopting a large value of the Young's modulus. Figure 1 also shows the properties of the homogeneous halfspace.

A 20×20 mesh of identical 8-node solid elements is used to simulate the foundation. This mesh leads to a 20×20 regular discretization of the interaction surface. The time step, Δt , used for the calculation is equal to 1.8214×10^{-5} s, which is in accordance with (13).

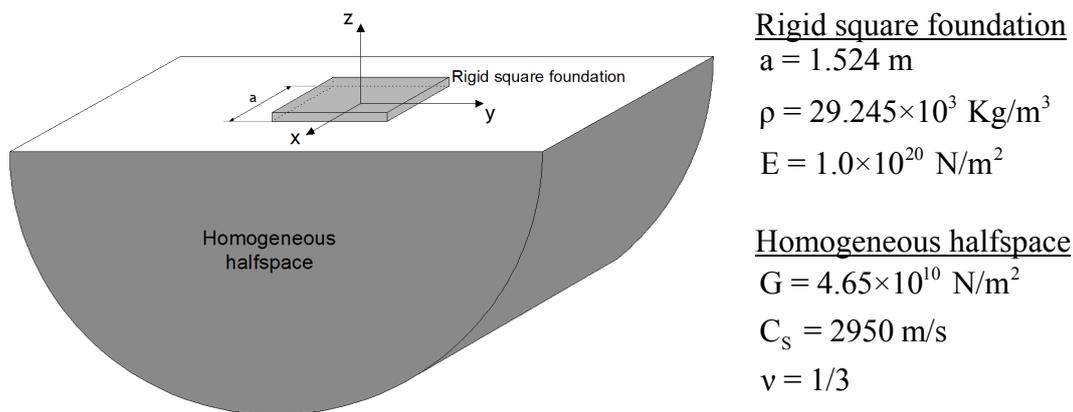


Figure 1: Geometry and properties of the foundation - soil system.

At a first stage the foundation is subjected to a short-duration vertical impulse, applied during one time step and distributed over its area. As a second phase of the study, a long-duration vertical impulse is applied during several time steps. Figure 2 illustrates the time history of each load.

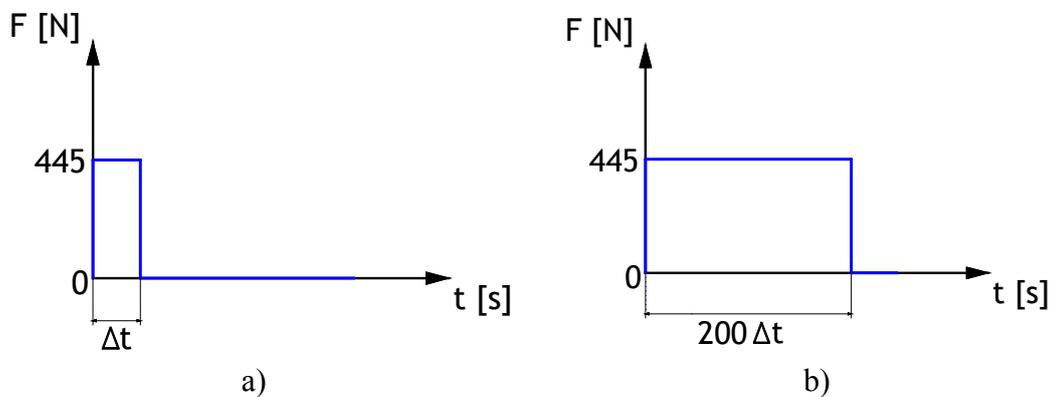


Figure 2: Time history of the applied loads: a) short-duration vertical impulse; b) long-duration vertical impulse.

Figures 3 and 4 show the time history of the vertical displacement of the rigid foundation subjected to a short-duration vertical impulse and to a long-duration one, respectively. A good agreement can be observed when comparing the results obtained in the present work with those presented in [7]. This fact indicates that the implemented method provides accurate results.

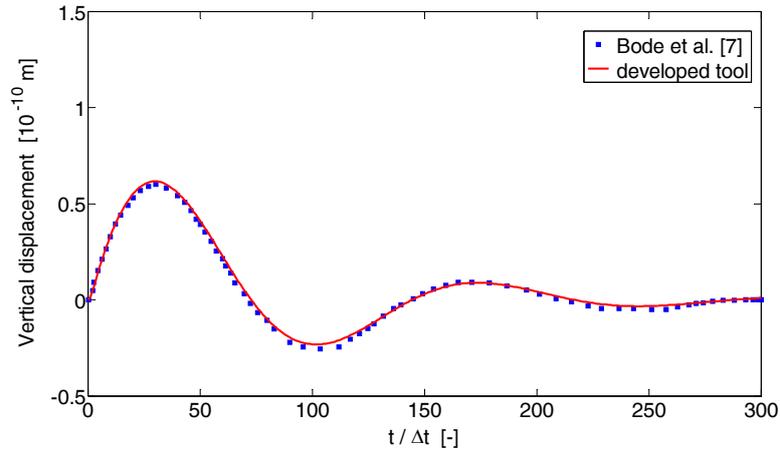


Figure 3: Vertical response of a rigid square foundation subjected to a short-duration vertical impulse (applied during one time step): comparison of the obtained results with those presented in [7].

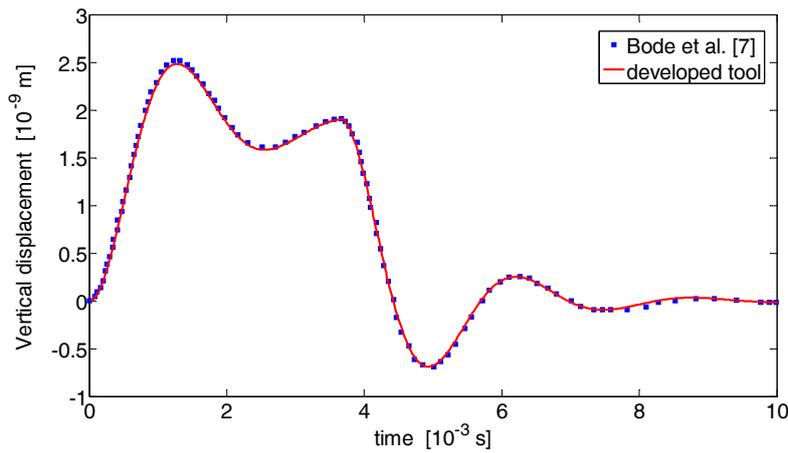


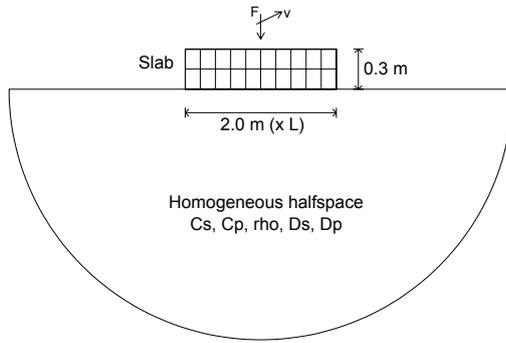
Figure 4: Vertical response of a rigid square foundation subjected to a long-duration vertical impulse (applied during 200 time steps): comparison of the obtained results with those presented in [7].

4.2 Load moving over a surface of a slab

As a second example, the dynamic response of a slab subjected to a moving load is studied. The aim of this study is to analyse the performance of the developed tool in the computation of the response of a structure subjected to a moving load. The obtained results are compared with those computed by means of a 2.5D FEM/ITM methodology [5, 13].

Figure 5 shows the geometry and the properties of the slab as well as the properties of the homogeneous halfspace. Different lengths of the model, L , were

considered in the calculation. The results presented in this paper correspond to a length L equal to 50 m. The slab is discretized into two layers of 8×200 identical 20-node solid elements with dimensions $0.25 \times 0.25 \times 0.15$ cubic meters. The chosen mesh leads to a 8×200 regular discretization of the interaction surface. Based on (13), a time step, Δt , equal to 1×10^{-3} s is adopted.



Slab

$E = 30 \times 10^9$ Pa
 $\rho = 2145$ Kg/m³
 $\nu = 0.2$

Homogeneous halfspace

$C_p = 433.0$ m/s
 $C_s = 250.0$ m/s
 $\rho = 1800$ Kg/m³
 $D_p = D_s = 0.02$

Figure 5: Geometry and properties of the slab - soil system.

A vertical load of magnitude F equal to 1 kN moving over the length of the slab, L , at different speeds, v , is considered in this study. The presented results correspond to a circulating speed equal to 125 m/s, which is below the velocity of the surface waves of the soil.

Figure 6 shows the evolution of the vertical displacement of a point on the top surface of the slab (corresponding to a node of the FE mesh at the mean section of the model: position equal to zero) with the load position. This figure shows a comparison between the obtained results with those calculated using a 2.5D FEM/ITM model. A good agreement between the results of both models can be observed. However it is possible to identify a small disturbance in the curve obtained in the present work, which occurs when the load enters the model.

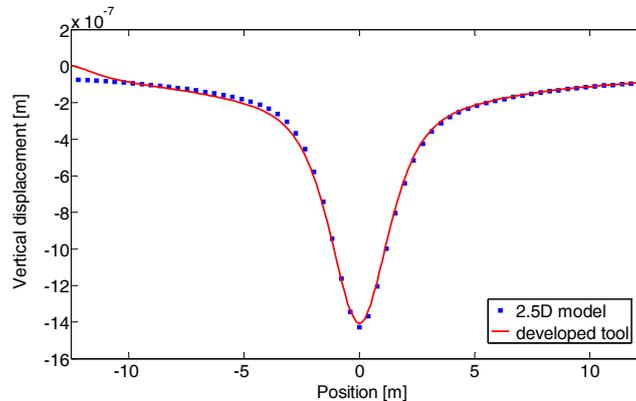


Figure 6: Vertical displacement of a point of the slab: comparison of the obtained results with those computed using a 2.5D FEM/ITM model.

In a similar representation, Figures 7 and 8 show the evolution of the vertical displacement of two points located 2 m and 5 m off the border of the slab, respectively. Once again a good agreement between the results is observed. However, the disturbance due to the entrance of the load in the model is more evident as the distance between the analysed point and the slab increases. This fact enables to conclude that the study of the response at larger distances from the slab requires a more extensive model.

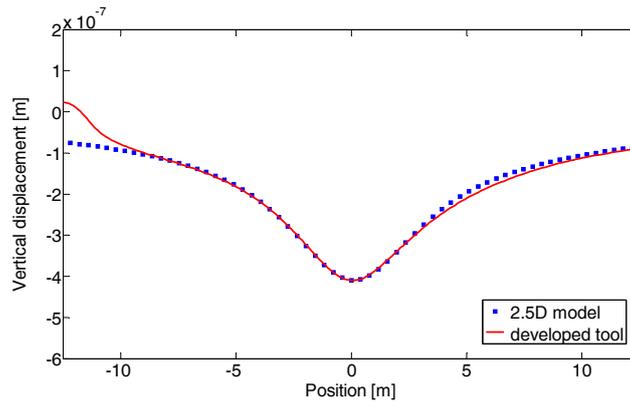


Figure 7: Vertical displacement of a point 2 m off the border of the slab: comparison between the obtained results and those computed with a 2.5D FEM/ITM model.

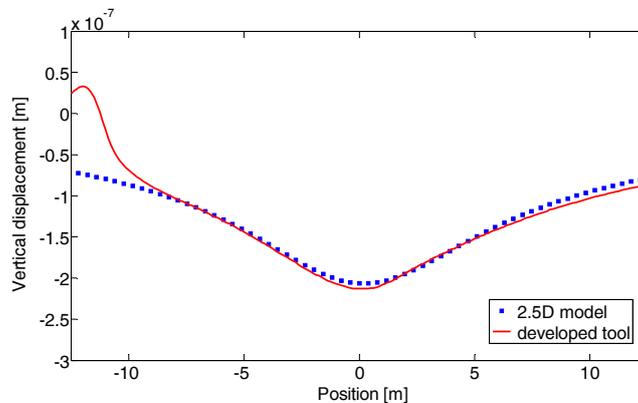


Figure 8: Vertical displacement of a point 5 m off the border of the slab: comparison between the obtained results and those computed with a 2.5D FEM/ITM model.

As an example, Figure 9 shows two colour plots of the vertical displacements of the slab at an intermediate instant of the analysis. In order to emphasise the effect of the load speed, the results due to a load moving at subcritical speed are compared to those obtained for the case of a load moving at supercritical speed. It is possible to observe that, when the load speed is below the velocity of the surface waves, displacements are practically symmetric. The symmetry disappears for the case of a supercritical load speed and the disturbance lies behind the load.

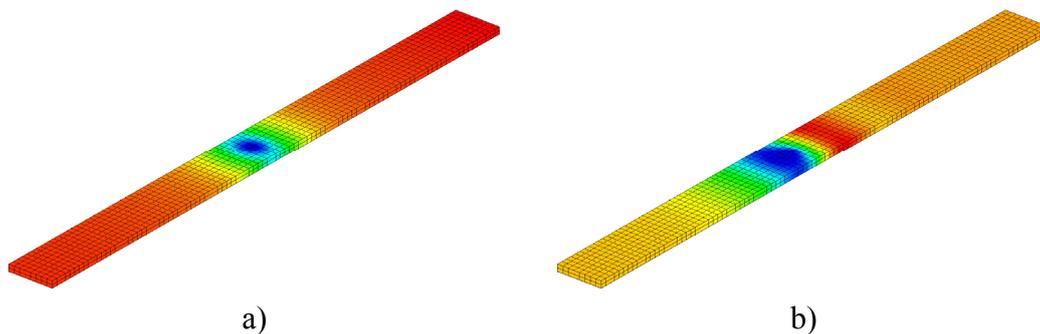


Figure 9: Vertical displacements of the slab showing the effect of the load speed:
a) subcritical speed; b) supercritical speed.

5 Conclusions and further developments

This paper briefly presents the implementation and verification of a numerical analysis methodology, which can be used in the prediction of vehicle induced vibrations. Two numerical examples are presented, which are included in a large group of tests related to the validation of the methodology. The comparison of the obtained results with those presented in the literature as well as with those obtained by means of a different validated method reveals a good agreement. However, since this tool was developed in order to be used in the prediction of vehicle induced vibrations, some tests with increasing complexity are still needed until that goal is achieved.

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