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Second-order Structural Optimization

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GENERAL PURPOSE OPTIMIZATION METHOD

- Large scale optimization (> 1000 design variables)
- Increased precision and reliability
- Second-order method

NONLINEAR PROGRAMMING

$$\text{Minimize } f\left(\underset{\sim}{x}\right)$$

$$\text{subject to}$$

$$\underset{\sim}{g}\left(\underset{\sim}{x}\right) \le \underset{\sim}{0} \quad \rightarrow \quad g_i\left(\underset{\sim}{x}\right) + s_i^2 = 0$$

$$\underset{\sim}{h}\left(\underset{\sim}{x}\right) = \underset{\sim}{0}$$

- Variables / functions ➡ real and continuous
- Symbolic manipulation of generalized polynomials

  Ex. $f\left(\underset{\sim}{x}\right) = 5.9 x_1^2 x_4^{-3} - 3.1 x_2 + 2.7 x_1^{-1} x_3 x_5^2 - 1.8$
- Straightforward derivation and evaluation

- Lagrangian: $L\left(\underset{\sim}{X}\right) = f\left(\underset{\sim}{x}\right) + \sum_{k=1}^{m} \lambda_k^g \left[g_k\left(\underset{\sim}{x}\right) + s_k^2 \right] + \sum_{k=1}^{p} \lambda_k^h h_k\left(\underset{\sim}{x}\right)$

- Variables: $\underset{\sim}{X} = \left(\underset{\sim}{s}, \underset{\sim}{\lambda}^g, \underset{\sim}{x}, \underset{\sim}{\lambda}^h\right)$

- Stationary point of the Lagrangian:

  system of nonlinear equations

$$\nabla L\left(\underset{\sim}{X}\right) = \underset{\sim}{0} \quad \Rightarrow \quad \left| \begin{array}{ll} 2 s_i \lambda_i^g = 0 & (i = 1, \ldots, m) \\ g_i + s_i^2 = 0 & (i = 1, \ldots, m) \\ \dfrac{\partial f}{\partial x_i} + \sum_{k=1}^{m} \lambda_k^g \dfrac{\partial g_k}{\partial x_i} + \sum_{k=1}^{p} \lambda_k^h \dfrac{\partial h_k}{\partial x_i} = 0 & (i = 1, \ldots, n) \\ h_i = 0 & (i = 1, \ldots, p) \end{array} \right.$$

- Lagrange-Newton method: $\underset{\sim}{H}\left(\underset{\sim}{X}^{q-1}\right)\Delta\underset{\sim}{X}^{q}+\nabla L\left(\underset{\sim}{X}^{q-1}\right)=\underset{\sim}{0}$

$$\underset{\sim}{H}= \begin{array}{c|c|c|c|c|} & (m) & (m) & (n) & (p) \\ \hline (m) & Diag\left(2\lambda_i^g\right) & Diag\left(2s_i\right) & \underset{\sim}{0} & \underset{\sim}{0} \\ \hline (m) & & \underset{\sim}{0} & \dfrac{\partial g_i}{\partial x_j} & \underset{\sim}{0} \\ \hline (n) & & & * & \dfrac{\partial h_j}{\partial x_i} \\ \hline (p) & \textit{SYMMETRIC} & & & \underset{\sim}{0} \\ \hline \end{array}$$

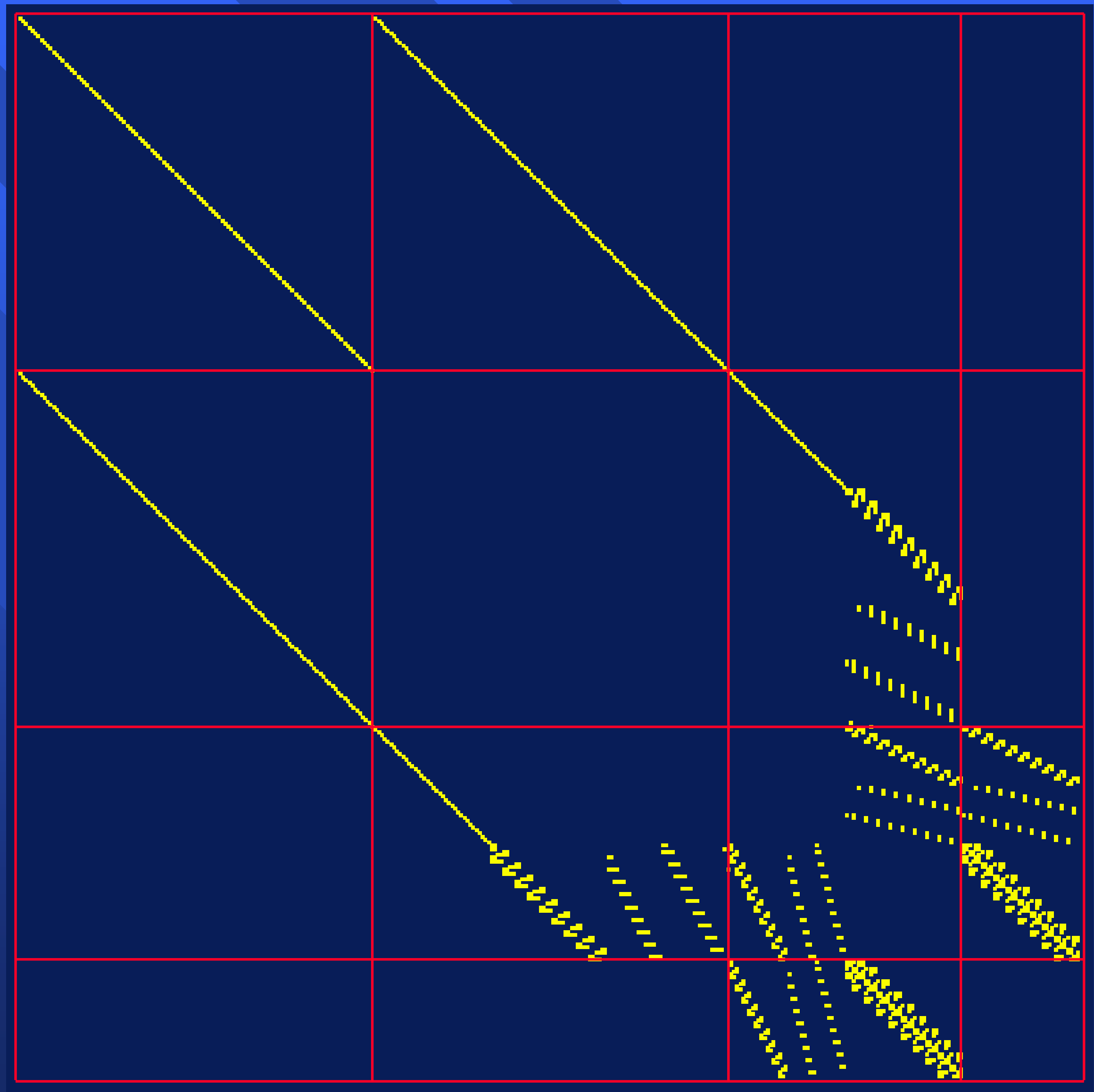
$$* \quad \frac{\partial^2 f}{\partial x_i \partial x_j}+\sum_{k=1}^{m}\lambda_k^g\frac{\partial^2 g_k}{\partial x_i \partial x_j}+\sum_{k=1}^{p}\lambda_k^h\frac{\partial^2 h_k}{\partial x_i \partial x_j}$$

HESSIAN MATRIX SPARSITY PATTERN

- Gaussian elimination

- Conj. grad. method:

$$\underset{\sim}{H}^{T} \cdot \underset{\sim}{H} \cdot \Delta \underset{\sim}{X} + \underset{\sim}{H}^{T} \cdot \nabla L = \underset{\sim}{0}$$



Gaussian elimination

- faster
- more reliable
- small pivots avoided
- RAM requirements increase considerably with the number of variables

Conjugate gradient method

- huge number of iterations
- too slow in large problems
- small RAM requirements

- Automatic
  - scaling of all the variables $(x_i = Z_i \, \bar{x}_i)$
  - normalization of the constraints
  - substitution of elementary eq. constraints
  - simplification of the nonlinear program

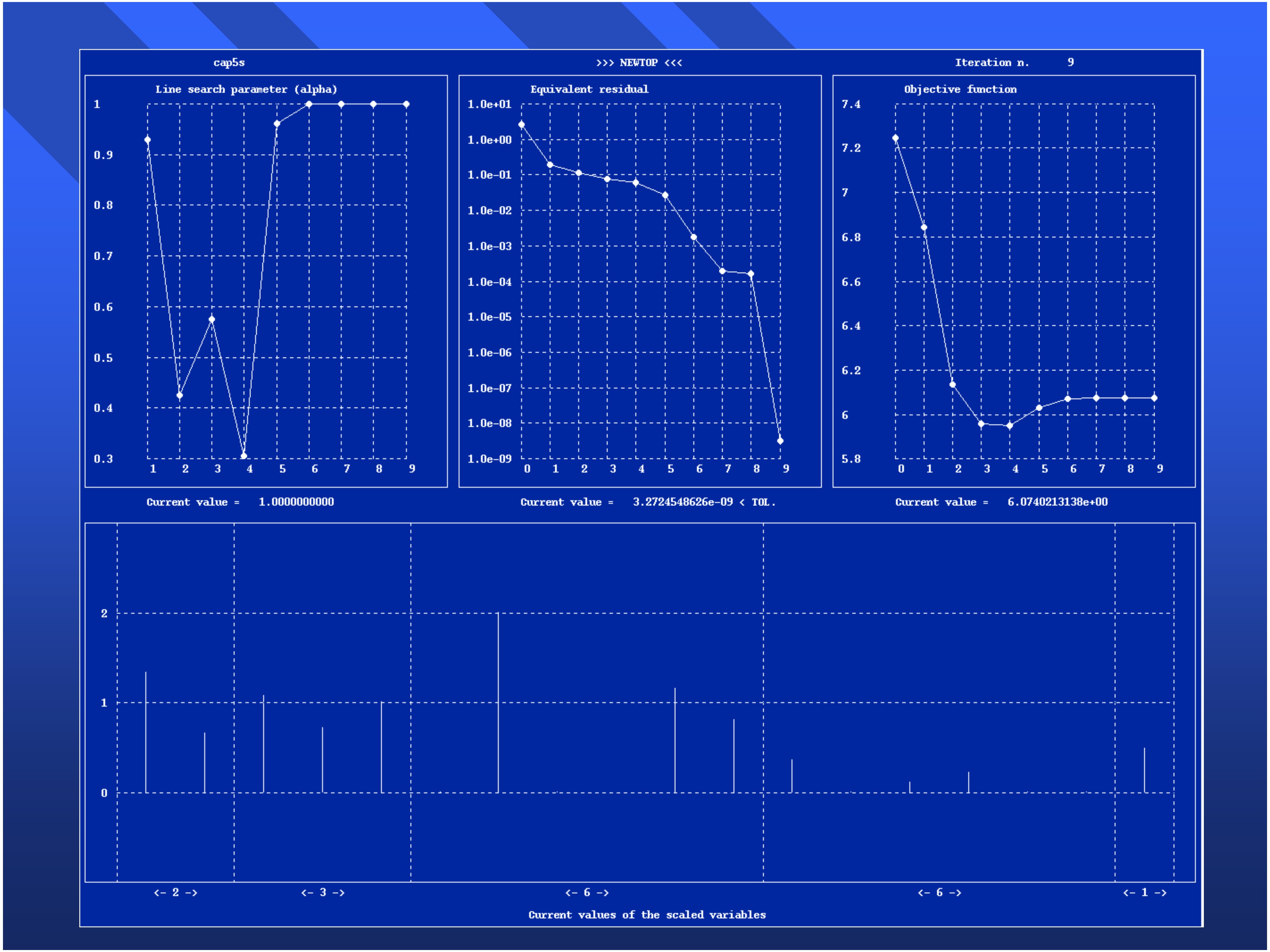
- Solution of the original NLP can be recovered

- Line search

NEWTOP COMPUTER PROGRAM (ANSI C)

- Input example:

```
### Main title of the nonlinear program
     Symmetric truss with two load cases (kN,cm)
Min.
     +565.685*t5^2 + 100*t8^2; # truss volume (cm3)
s.t.i.c.
     Min.area 4: -t4^2 + 0.15 < 0;
s.t.e.c.
     Equil.16: +141.421*t5^2*disp16 - 100 = 0;
END_OF_FILE
```


cap5s
>>> NEWTOP <<<
Iteration n. 9
Line search parameter (alpha)
Equivalent residual
Objective function
Current value = 1.0000000000
Current value = 3.2724548626e-09 < TOL.
Current value = 6.0740213138e+00
<- 2 ->
<- 3 ->
<- 6 ->
<- 6 ->
<- 1 ->
Current values of the scaled variables

STRUCTURAL OPTIMIZATION

- Integrated formulation
- In large scale problems the following transformation is advantageous:

$$\mathbf{h} = \mathbf{0} \rightarrow \left| \begin{array}{l} \mathbf{h} \leq \mathbf{0} \\ -\mathbf{h} \leq \mathbf{0} \end{array} \right.$$

- Truss sizing examples:
  - stress, displacement and side constraints
  - one load case

- Desktop workstation: 256 MB RAM; 40 MFlops

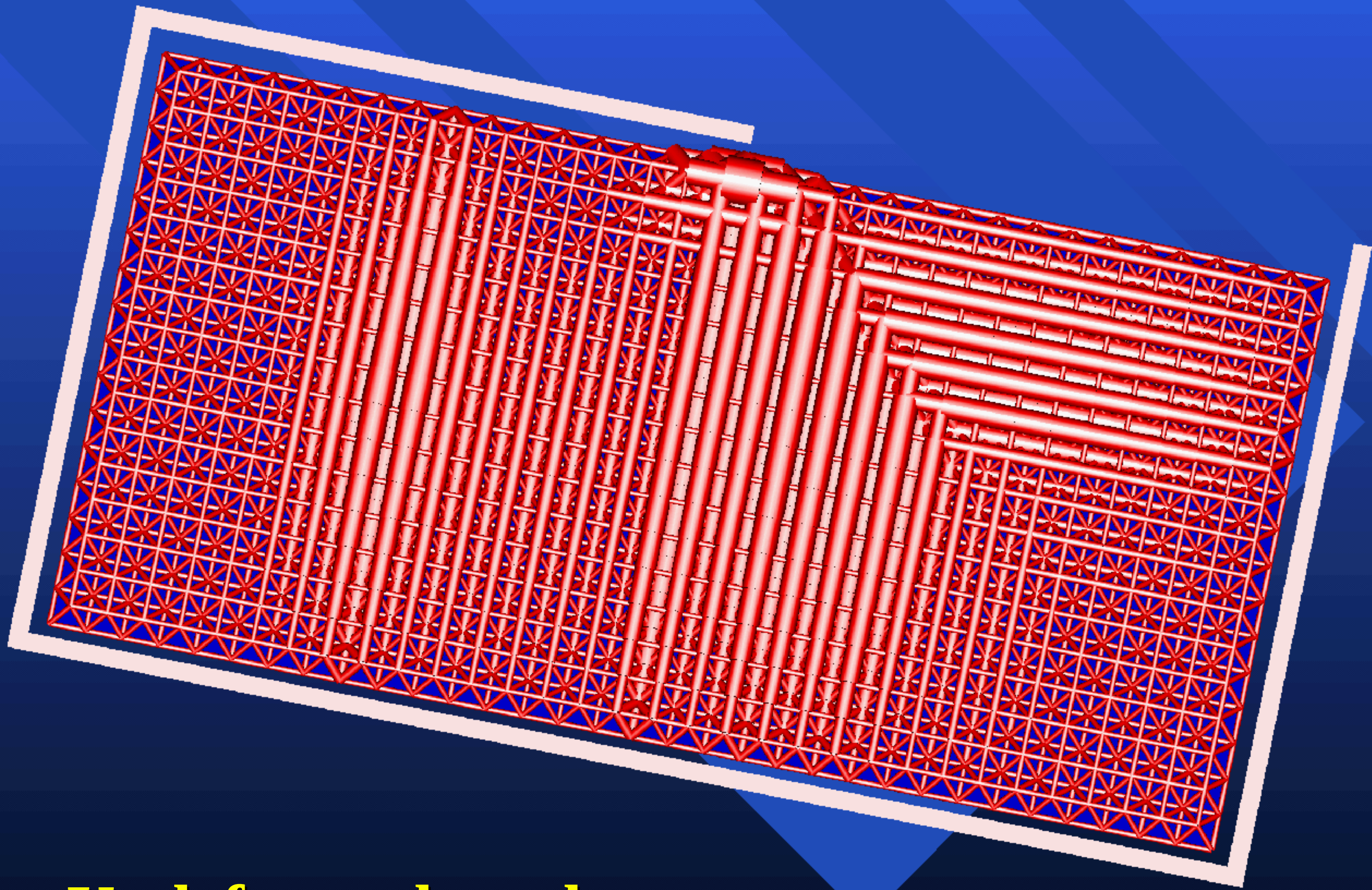
- Computation time:
  - small problems (100 bars) → a few seconds
  - medium problems (1000 bars) → a few hours
  - large problems (4000 bars) → a few days

LARGE SCALE OPTIMIZATION EXAMPLE

3D truss sizing

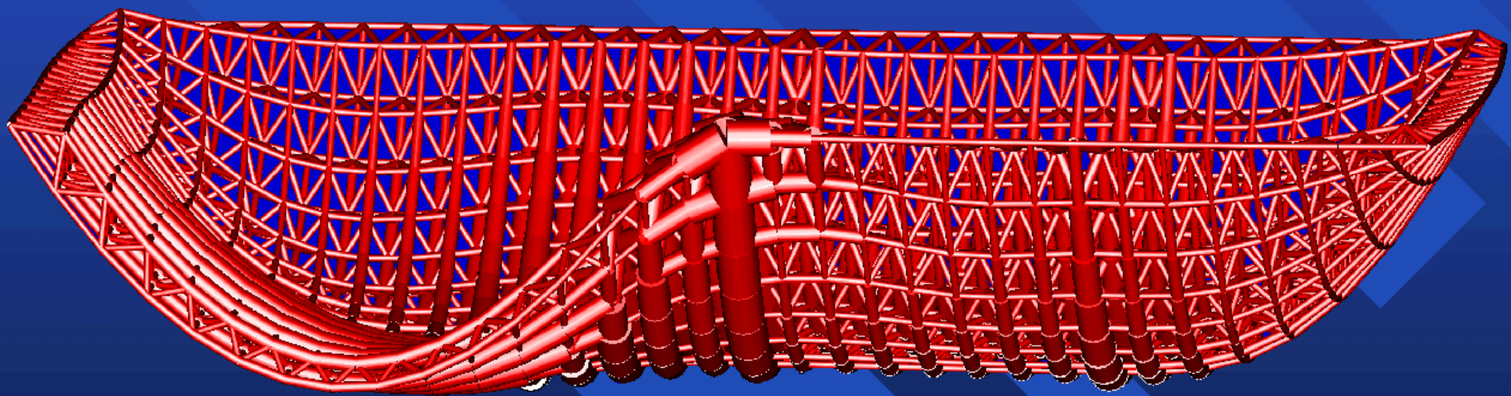
- Number of bars = 4 096
- Number of degrees of freedom = 3 135
- Number of decision variables = 7 231
- Number of inequality constraints = 19 038
- No variable linking; no active set strategy

BUILDING ROOF - OPTIMAL SOLUTION



Undeformed mesh

BUILDING ROOF - OPTIMAL SOLUTION



Deformed mesh

NEWTOP ALGORITHM

➡ ADVANTAGES

- PRECISION
- VERSATILITY
- RELIABILITY
- CAPACITY

NEWTOP ALGORITHM

➡ **DRAWBACKS**

- **EFFICIENCY ?**

- **INTEGRATED FORMULATION**

Too demanding when the *n. design variables* is small

and the *n. load cases* x *n. degrees of freedom* is high